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RESEARCHING AND MODELING OF THE DYNAMIC VISCOSITY OF THE FERRO-OILS WITH THE DIFFERENT CONCENTRATIONS OF MAGNETIC PARTICLES IN THE ASPECT OF PRESSURE CHANGES

BADANIA I MODELOWANIE LEPKOŚCI DYNAMICZNEJ FERRO-OLEJÓW O RÓŻNYM STĘŻENIU CZĄSTEK MAGNETYCZNYCH W ASPEKCIE ZMIAN CIŚNIENIA

Key words: ferro-oil, magnetic particle's concentration, pressure changes, models of viscosity.

Abstract: In this article, the results of the experimental research of determining the dynamic viscosity of the ferro-oils with different concentration of the magnetic particles in terms of pressure change are presented. The research was conducted on the Thermo Scientific Haake Mars III rheometer in the configuration with a *pressure chamber*. The applied configuration allowed the author to conduct research on the change of viscosity for the selected pressure values p in range from 0 to 100 bars. The research was conducted for three selected temperature values, including t = 30°C, 60°C, and 90°C, as well as selected shear rates of 200 and 600s^{-1}. The chosen concentration values of the magnetic particles were n_{CS} = 1%, 2%, 4%, 6%, and 8%. The viscous characteristics gathered as a result of the research have been analysed with the aim of identifying the nature of the correlations and to create mathematical-physical models of changes in the dynamic viscosity of the ferro-oils due to the change in pressure in terms of concentrations of the magnetic particles. Analysis, identification, and adaptation of the characteristics were conducted using the StatSoft STATISTICA ver. 9 software. Selected mathematical adaptations of the acquired results were submitted through the enhanced analysis. In addition, parameters of the course of functions were designated and their qualitative assessments were made.

Słowa kluczowe: ferro-olej, stężenie cząstek magnetycznych, zmiany ciśnienia, modele lepkości.

Streszczenie: W artykule zaprezentowane zostały wyniki eksperymentalnych badań wyznaczania lepkości dynamicznej ferro-olejów o różnym stężeniu cząstek magnetycznych w aspekcie zmian ciśnienia. Badania zostały przeprowadzone na reometrze Thermo Scientific Haake Mars III w konfiguracji z tzw. *komorą ciśnieniową*. Zastosowana konfiguracja pozwalała na prowadzenie badań zmian lepkości dla wybranych wartości ciśnienia p z zakresu od 0 do 100 bar. Badania przeprowadzono dla trzech wybranych wartości temperatur dla t = 30°C, 60°C oraz 90°C i wybranych prędkości ścinania: 200 oraz 600 s^{-1}. Przyjęte wartości stężenia cząstek magnetycznych wynosiły n_{CS} = 1%, 2%, 4%, 6% i 8%. Uzyskane w wyniku badań charakterystyki lepkościowe poddane zostały analizie zmierzającej do identyfikacji charakteru zależności i wykreowanie matematyczno-fizycznych modeli zmian lepkości dynamicznej ferro-olejów od zmian ciśnienia w aspekcie stężenia cząstek magnetycznych. Analizy, identyfikacji i dopasowania charakterystyk dokonano przy wykorzystaniu oprogramowania StatSoft STATISTICA wer. 9. Pogłębionej analizie poddano wyselekcjonowane dopasowania matematyczne uzyskanych wyników doświadczalnych. Wyznaczono ponadto parametry przebiegu funkcji i dokonano ich oceny jakościowej.

INTRODUCTION

The issues addressed in the following work, in its broader approach, refer to the research project regarding the analysis of change in the flow and operational parameters of the journal slide bearings lubricated with ferro-oils characterized by different concentrations of magnetic particles.

When deciding on the use of a particular oil, including ferro-oil, in the process of the lubrication

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of slide bearings, numerous features are taken into consideration, including the choice of rheological properties and, in particular, its viscosity. This property is mainly dependant on the basic parameters of work – temperature, pressure, and shear rate. Other significant parameters include the oil's chemical composition as well as the internal structure expressed, among others, in the concentration of the magnetic particles ncs. In addition, in the case of ferro-oils, the presence and characteristics of the external magnetic field, varying depending on the type, direction or the induction value, play the key role in their viscosity.

The dynamic viscosity of the lubricating oil directly influences the operational parameters of the sidle bearings, such as friction force, load carrying capacity, and the coefficient of friction. The values of these parameters are the evidence of the work quality of slide friction nodes. Conducting further research specifying the influence, both quantitative and qualitative, of the concentration of the magnetic particles on the change in the physical properties of the ferro-oils and, in consequence, tribological properties seems to be essential in this context. The rules of the selection of the optimal concentration of the aforementioned particles, depending on the existing environmental conditions, its behavior, or particular expectations for the work of the equipment, need to be specified.

The aim of this work is to identify and indicate rheological characteristics of the ferro-oils in the context of pressure change, their description and comparison.

THEORETICAL FOUNDATIONS

In the previous works presented by the author in numerous publications, experimental research has been conducted referring to the influence of the magnetic particle concentrations on the viscosity properties of the ferro-oil, both in the presence of [**L. 1, 2**] as well as without the presence [**L. 3, 4**] of an external magnetic field in terms of the change of the basic work parameters of the potential bearing, i.e. temperature, pressure, and shear rate $\eta = \eta(T, p, \theta)$. Furthermore, coefficients of the magnetic susceptibility χ of the ferro-oil have also been designated through experiments, in correlation with the concentration of magnetic particles [**L. 5**]. Acquired results, observations made and conclusions, both qualitative and quantitative, did not conclusively indicate the mathematical-physical form of the relationship between the researched quantities and the concentration of the magnetic particles in the ferro-oil.

The current stage of research requires performing the adjustment of the mathematical models of change in dynamic viscosity of the ferro-oil depending on the temperature, pressure, and the intensity of the external magnetic field, as well as the calculation of the coefficients of change in such viscosity depending on the listed parameters: δT, δp, δB. In this work, an appropriate analysis has already been conducted as well as calculations of the parameters δB specifying the relationship of the dynamic viscosity of the ferro-oils in the context of the change in the intensity of the magnetic field $\eta = \eta(B)$. In works [**L. 7, 8**], accordingly, an analogical analysis was made regarding the relation of $\eta = \eta(T)$ and the calculation of the parameters δT in the context of change in viscosity, depending on the temperature. The following publication constitutes a continuation of previous research studies and presents the proposition of the selection and adjustment of the appropriate functions regarding the specific relation between the dynamic viscosity of the researched ferro-oil of chosen concentrations of magnetic particles with changes in pressure $\eta = \eta(p)$ and calculations of parameter values δp of these functions.

A performed overview of solutions proposed in the literature on the subject can be brought to a few simple models.

The most general of the models can be found, among others, in works [**L. 9, 10, 11**]. The authors quoted propose to describe the function of the correlation between the dynamic viscosity and temperature $\eta(p, T)$ using the generalized Roelands' equation in the following form:

$$\eta_c(p,T) = \eta co \cdot \exp[f(\eta_{co}, p, T, T_0, z, s_o)] \qquad (1)$$

where f – Roeland's function obtained in experimental way [**L. 8**],
η_c – dynamic viscosity of ferrofluid [Pa·s],
η_{co} – initial dynamic viscosity at standard conditions of pressure and temperature [Pa·s],
p – pressure in the oil film [Pa],
T – temperature in the oil film [K],
T_0 – reference temperature T_0 = 29315 K,
s_o – temperature coefficient of viscosity s_o = 1.1,
z – pressure coefficient of viscosity z = 0.68.

When analyzing the above mentioned model, a few important facts should be noted. First of all, the mathematical complexity of the model is non-negligible, which substantially impacts its practical application possibilities during further analytical-numerical research. It is possible to distinguish parts of entirely different mathematical constructions: logarithmic, exponential, homographic, as well as polynomial. Secondly, acquired coefficients constituting the above mentioned dependence have empirical origin, and the possibilities of applying this formula are restricted to fluids of similar characteristics. Thirdly, a significant fault of the model is the lack of parametric references including the internal structure of the ferro-fluids, with the particular inclusion of the concentrations of magnetic particles ncs. Only the parameters describing the work conditions of the fluid, i.e. pressure and temperature, were included in the construction of this model. In the enhanced version of the above mentioned model, the influence of the external magnetic field B is additionally taken into account by adding the correction part.

$$\eta_f(p, T, H) = \eta_{f0} + k_1 \times \Delta\eta(H) \quad (2)$$

where ηf – dynamic viscosity of the ferrofluid in an external magnetic field [Pa·s],
η_{f0} – dynamic viscosity of the ferrofluid without the magnetic field [Pa·s],
k_1 – proportionality coefficient obtained experimentally,
$\Delta\eta$ – increase in dynamic viscosity under the influence of external magnetic field [Pa·s].

In the works of A.Miszczak and K.Wierzcholski, among others, [**L. 12, 13**], a much more "friendly" analytical-numerical research dependence model $\eta = \eta(T, p, B)$ has been described. The dynamic viscosity of the ferro-oil has been written as the product of parts dependent on the pressure, temperature, as well as the external magnetic field as follows:

$$\eta_1(B, p, T) = \eta_{1B} \cdot \eta_{1p} \cdot \eta_{1T} \quad (3)$$

where each of the main units has been appropriately modeled by the exponential function

$$\eta_{1B}(\phi, z) = \exp(\delta B \cdot B_0 \cdot B_1) = \exp(\delta B_1 \cdot B_1) \quad (4)$$

$$\eta_{1p}(\phi, z) = \exp(\zeta \cdot p_0 \cdot p_1) = \exp(\zeta_p \cdot p_1) \quad (5)$$

$$\eta_{1T}(\phi, z, r) = \exp(-\delta T \cdot (T-T_0)) = \exp(-Q_{Br} \cdot T_1) \quad (6)$$

Where: $\eta_1(B, p, T)$ – the dimensionless dynamic viscosity depends on the magnetic induction, pressure and temperature,
$\eta_{1B}(\phi, z)$ – dimensionless dynamic viscosity depends on the magnetic induction,
$\eta_{1p}(\varphi, z)$ – dimensionless dynamic viscosity depends on the pressure,
$\eta_{1T}(\varphi, z, r)$ – dimensionless dynamic viscosity depends on the temperature,
δ_B – dimensional coefficient taking into account the effect of the magnetic induction B changes on the dynamic viscosity [T^{-1}],
δ_{B1} – dimensionless coefficient taking into account the effect of the magnetic induction B changes on the dynamic viscosity,
B_0 – magnetic induction [T],
B_1 – dimensionless magnetic induction,
ζ – dimensional piezocoefficient of the viscosity [Pa^{-1}],
ζp – dimensionless piezocoefficient of the viscosity,
δ_T – dimensional coefficient taking into account the effect of the temperature T changes on the dynamic viscosity [K^{-1}],
Q_{Br} – dimensionless coefficient of viscosity changes depends on temperature.

The above mentioned model is characterized by high susceptibility to the possibility of its application in further analytical and numerical research. Unfortunately, this advantage came at the price of the inadequacy of the model in relation of the actual nature of the described phenomena. These discrepancies are particularly strongly affirmed in the case of pressure η_{1p} and magnetic η_{1B} parts. The authors of work [**L. 6**] indicated the aforementioned mentioned inadequacy for the magnetic part and proposed an alternate solution.

The authors of works [**L. 14, 15**] propose to describe the dependency of change in the dynamic viscosity of the mineral oils with the Barus's equation [**L. 16**]:

$$\eta = \eta_0 \cdot \exp(\alpha \cdot p) \quad (7)$$

where η_0 – characteristic value of dynamic viscosity coefficient for $T_0 = 273{,}15$ K i $p_0 = p_{at}$ [Pa·s],
α – material constant of oil [Pa^{-1}].

On the one hand, this dependency is characterized by a simple mathematical construction, which consequently has a high susceptibility to the possibility of its application in further analytical and numerical research being a non-negligible advantage of this model. On the other hand, its fault lies in the lack of parametric references, including the internal structure of the ferro-oil affirmed in the inclusion of the concentration of the magnetic particles ncs in the mathematical and physical construction of the model. It has been proved in [**L. 4**] that the exponential model of the change in the viscosity with pressure is may be used exclusively for the values of pressure not exceeding 5 bars, both in accordance with the conclusions resulting from the Barus's dependence as well as those obtained in my own results acquired in the process of empirical research. Application of the exponential model for the description of change in pressure above 5 bars can result in the inadequacy of the model in relation to the actual processes, and in effect, can generate substantial calculation errors during further stages of the research. Another fault of this model is the lack of parametric reference in relation to the concentration of magnetic particles included within it.

It is also important to note that, despite the fact that the addition of magnetic particles significantly changes the microstructure and properties of the mineral oil, it is does not influence the nature of the dynamic viscosity curve in relation to the pressure for the ferro-oils and base oils, which indicate a significant similarity. In the course of the work, the author indicates the alternative ways of modeling the above dependencies and makes their assessment.

METHODOLOGY OF RESEARCH

As a result of the conducted research, measurements of the dynamic viscosity have been made for the ferro-oils that are the colloidal mixture of the LongLife Gold mineral oil manufactured by the Pennzoil Company, magnetic particles of the Fe3O4 iron oxide and the surfactant. The researched ferro-oil has been manufactured by the FerroTec Company from Unterensingen (Germany). The percentage share (volume) of the magnetic particles in the researched samples was 8%, 6%, 4%, 2%, and 1%, accordingly, and their average diameter was 10 nm. Volume content of the unspecified name surfactant was about 15%. Original, delivered by the manufacturer, percentage concentrations of the magnetic particles in the researched ferro-oils were 6% and 8%. Other researched concentrations, i.e. 4%, 2% and 1%, were obtained by the author as a result of dilution of the original samples with the above mentioned base oil and a replacement surfactant which was a 25% mixture of hydroxide of tetramethylammonium hydroxide $(CH_3)_4NOH$.

Table 1 below presents the values of the physical parameters accepted in the research: pressures, temperatures, and shear rates.

Table 1. Range of physical parameters used in tests of viscosity depended on pressure without the presence of an external magnetic field

Tabela 1. Zakresy liczbowe parametrów fizycznych zastosowane w badaniach lepkości od ciśnienia bez obecności zewnętrznego pola magnetycznego

Magnetic particle concentrations in ferro-oil	Geometry of rotors	Adopted pressure values	Adopted shear rate values	Adopted temperature values
1%, 2%, 4%, 6%, 8% and base oil (0%)	"cylinder-cylinder"	0, 1,10, 40, 70, 100 [bar]	200, 600 [s^{-1}]	30, 60, and 90 [°C]

The research on the rheological properties of the ferro-oil and base oil samples in terms of pressure change, without the presence of the external magnetic field, were conducted on the rotating rheometer HAAKE MARS III configured in the setting with the pressure chamber, allowing for applying pressure changes up to 100bars. In geometric terms, a setting of a "cylinder-cylidner" type was used with the PZ38 rotor. The measurements were conducted in the CR mode (Constant Rotation) in which an operator applies a constant rotation value of the rotor and torque is measured. A drive was transmitted to the rotor through the magnetic coupling from the main rotation driver of the rheometer. Control tests with the hall-effect sensor conducted by the author indicated that the influence of the magnetic coupling does not transfer to the research pressure chamber of the device and therefore has no magnetic influence on the rheological properties of the ferro-oil samples.

The accuracy of the acquired results, in the case of the research in the context of pressure change, was strongly dependent on the appropriate preparation of the measuring device. Firstly, it was necessary to consider the influence of the change in temperature on the magnetic coupling of the rotor with the drive elements and to designate the appropriate gap between the fixed magnets. In the next stage, it was necessary to consider the influence of the possible imprecision in the fitting of the rotor as well as the influence of the force of friction in its bearings. With the use of appropriate tests preparing the device for the measurement, it was necessary to specify the value of the friction force on the bearings "wetted" with the fluid identical to the one used in the researched sample.

It is worth mentioning that a singular measurement of the change in the viscosity in relation to the pressure required the volume of at least 50ml of the researched sample of the ferro-oil in order to fill the measurement chamber, and, due to the costs, the research was conducted only once for each of the courses.

Enhanced analysis was conducted for three selected mathematical adjustments for the acquired experimental results. Analysis, identification, and adjustment of the characteristics were made using the StatSoft STATISTICA ver.9 software. Gauss-Newton non-linear estimation was applied with regression performed using the method of the smallest squares, for the maximal adopted number of iterations equal 250, using the criterion of convergence equal 1×10^{-6}. The adopted level of trust was at p = 0.95.

ANALYSIS OF THE RESULTS

The illustrations below, **Figures 1** through **3**, present the acquired results referring to the change in the dynamic viscosity of ferro-oils and base oil in terms of pressure change, in relation to the concentration of the magnetic particles, prepared for the two selected values of shear rate (θ = 200s^{-1} and 600s^{-1}) as well as for the three selected temperature values t = 30°C, 60°C, and 90°C.

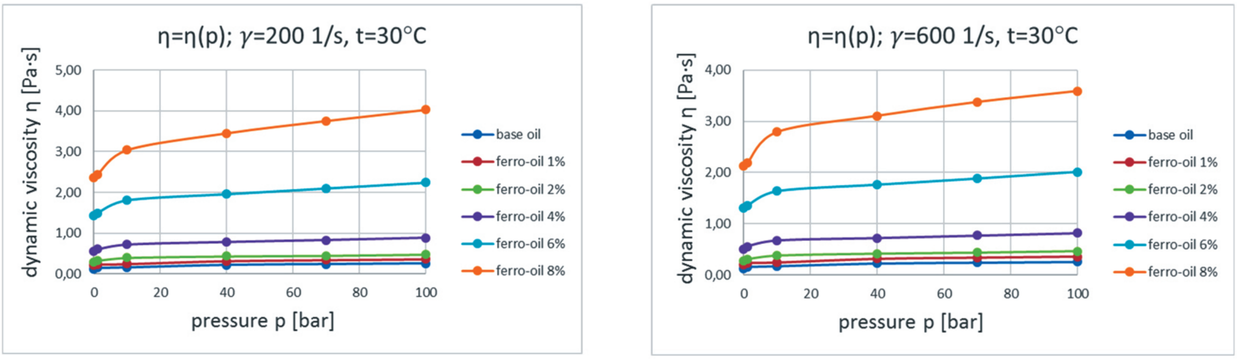

Fig. 1. Characteristics of changes in dynamic viscosity of ferro-oils and base oil with pressure for t = 30°C
Rys. 1. Charakterystyki zmian lepkości dynamicznej ferro-olejów i oleju bazowego od zmian ciśnienia dla t = 30°C

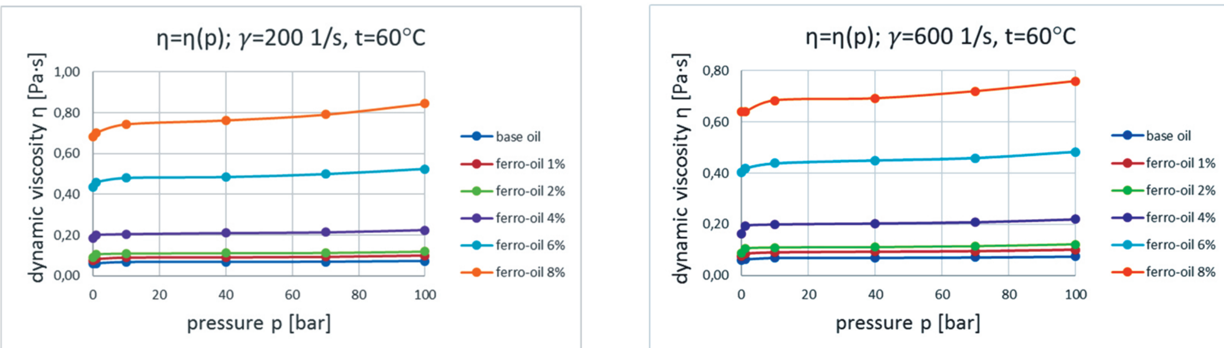

Fig. 2. Characteristics of changes in dynamic viscosity of ferro-oils and base oil with pressure for t = 60°C
Rys. 2. Charakterystyki zmian lepkości dynamicznej ferro-olejów i oleju bazowego od zmian ciśnienia dla t = 60°C

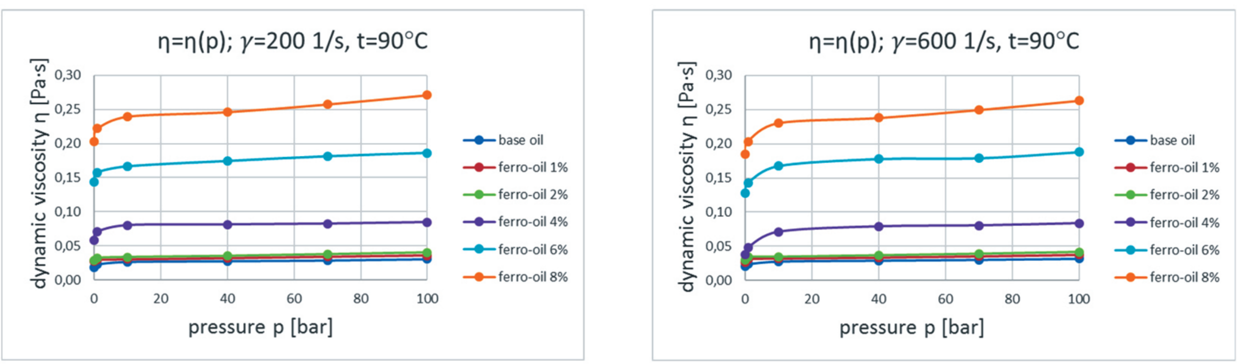

Fig. 3. Characteristics of changes in dynamic viscosity of ferro-oils and base oil with pressure for t = 90°C
Rys. 3. Charakterystyki zmian lepkości dynamicznej ferro-olejów i oleju bazowego od zmian ciśnienia dla t = 90°C

Analyses of the characteristics presented on the illustrations above indicate the important influence of change in pressure on the values of the dynamic viscosity of the ferro-oils. In the case of both low and high concentrated ferro-oils, the nature of the change in viscosity is similar in its course. This applied to the course of the characteristics for the base oil as well. Relative increases in the dynamic viscosity for all concentrations of the ferro-oils, in the researched range of pressure change, are included in the ranges between 56% and 72% for the lower temperatures (30°C) and slower shear rates ($200s^{-1}$) and between 29% and 44% for the higher temperatures (90°C) and faster shear rates ($600s^{-1}$). With the same temperature, the parameters and shear rate base oil indicated slightly higher relative increases in dynamic viscosity with the rise in pressure, up to 112% and 62%, accordingly.

Absolute increases in the dynamic viscosity were strictly correlated with the concentration of the magnetic particles in the ferro-oil. The larger the concentration was, the higher absolute increases in viscosity were, ranging up to 1.44 Pa·s for the 8% ferro-oil at the temperature of 30°C, regardless of the shear rate values.

Additionally, enhanced analysis of the results presented above was directed towards the identification of the nature of the dependencies and creation of the mathematical and physical models of the dynamic viscosity of the ferro-oils in relation to the pressure change in terms of the magnetic particles concentration.

The first of the conducted adjustments is the exponential model directed towards the identification of the dependency of the dynamic viscosity in correlation with the parameter of the magnetic particles concentration in ferro-oil as described in the referenced literature. The second proposed model is the logarithmic model, also indicating dependency of viscosity on the concentration of the magnetic particles. The third model is the Power model.

Model 1: Exponential adjustment (8), was described with the relation of the following form:

$$\eta_p = \eta_0 \cdot a^1 \cdot \exp(\delta_{p1} \cdot p) \quad (8)$$

where η_p – dynamic viscosity depended on pressure [Pa·s],
η_0 – initial dynamic viscosity for p_0 [Pa·s],
δ_{p1} – factor taking into account the influence of pressure p on the dynamic viscosity of the ferro-oil for the exponential model [Pa^{-1}],
a_1 – proportionality factor for exponential model [-],
p – hydrodynamic pressure [Pa].

Figure 4 below represents the obtained characteristics of the adjustment for the exponential model.

The adopted exponential function maps the experimentally obtained research results in a moderately satisfying degree. The share of the explained variance R2 is between 0.604 for the 2% ferro-oil and 0.712 for the 1% ferro-oil. A big advantage of Model 1 of exponential adjustment is the fact that the character of the change in the dynamic viscosity in relation to the pressure is analogical to the nature of change in the model of the adjustment of change in viscosity with temperature. The application of the model in this form would simplify further analytical and numerical research.

For Model 1 described with the Equation (8), an analysis of the parameters of adjustment has been made aiming to designate dependencies between them and the concentration of the magnetic particles in ferro-oil. The coefficient of proportionality a1 has been adjusted using the linear function, and the pressure parameter remained independent. The relation below (9) describes the obtained dependency.

$$\eta_p = \eta_0 \cdot (6{,}2728 \cdot ncs + 1{,}4211) \cdot \exp(\delta_{p1} \cdot p) \quad (9)$$

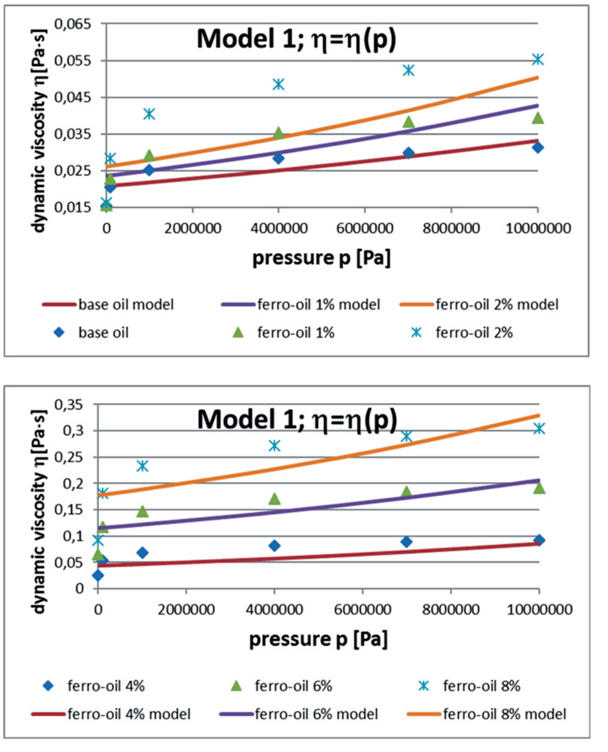

Fig. 4. Exponential model of function of dynamic viscosity changes with pressure for ferro-oils and base oil

Rys. 4. Model wykładniczy funkcji dopasowań zmian lepkości dynamicznej z ciśnieniem dla ferro-olejów i oleju bazowego

The obtained relation does not make the previous form of the function complicated (8), in a mathematical sense, and it can be successfully used in the further stages of the mathematical modeling, in analytical and numerical transformations.

Figure 5 presents the adjustments for the two extreme concentrations n_{cs} = 0% and n_{cs} = 8% estimated with this model.

The obtained values of the adjustment parameters as well as corresponding shares of the explained variations for the exponential model are presented in **Table 2**.

The second of the analyzed adjustment models, logarithmic, was described with the following relation (10):

$$\eta p = \eta_0 \cdot a_2 \cdot \ln(\delta_{p2} \cdot p) \quad (10)$$

where η_p – dynamic viscosity depended on pressure [Pa·s],
η_0 – initial dynamic viscosity for p_0 [Pa·s],
δ_{p2} – factor taking into account the influence of pressure p on the dynamic viscosity of the ferro-oil for logarithmic model [Pa^{-1}],
a_2 – proportionality factor for logarithmic model [-],
p – hydrodynamic pressure [Pa].

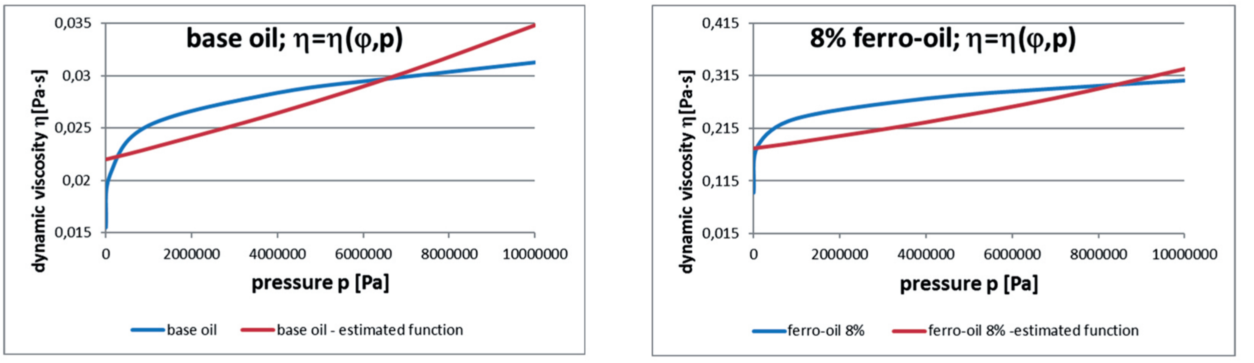

Fig. 5. Exponential model – fitted functions for the results of the viscosity changes due to the temperature for base-oil and 8% ferro-oil
Rys. 5. Model wykładniczy funkcji dopasowań – przykładowe zmiany lepkości dynamicznej z ciśnieniem dla 8% ferro-oleju oraz oleju bazowego

Table 2. The parameter values of matching function and the variance explained for Model 1
Tabela 2. Wartości parametrów funkcji dopasowania oraz wariancji wyjaśnionej dla Modelu 1

Magnetic particle concentrations	base oil	1% ferro-oil	2% ferro-oil	4% ferro-oil	6% ferro-oil	8% ferro-oil
R^2 – share of square variance	0.6978182	0.7121308	0.6045001	0.6427588	0.6423666	0.6266549
η_0 – characteristic value of dynamic viscosity	0.1547	0.01563	0.01940	0.02598	0.06551	0.09230
a_1 – proportionality ratio	1.352213	1.516317	1.605946	1.684889	1.755983	1.928807
δ_{p1} – pressure parameter	$4.594 \cdot 10^{-8}$	$5.889 \cdot 10^{-8}$	$6.531 \cdot 10^{-8}$	$6.640 \cdot 10^{-8}$	$5.834 \cdot 10^{-8}$	$6.149 \cdot 10^{-8}$

Figure 6 represents the characteristics of adjustments for the adopted logarithmic model (10) obtained with the use of dependencies.

The logarithmic model seems to reflect the nature of the change in viscosity in relation to the pressure to a much better degree than the previously analyzed exponential model. The values of the explained variance R2 range between 0.883 for the 8% ferro-oil and up to 0.994 for the 2% ferro-oil. Unfortunately, the disadvantage of this model is the occurrence of the discontinuity of the function in the "0" point, which most probably excludes the possibility of its application in the further analytical and numerical research due to the inability to indicate the values of viscosity at this point. In addition, this model maps a relatively weak area of low and very low pressures, generating significant deviations of values.

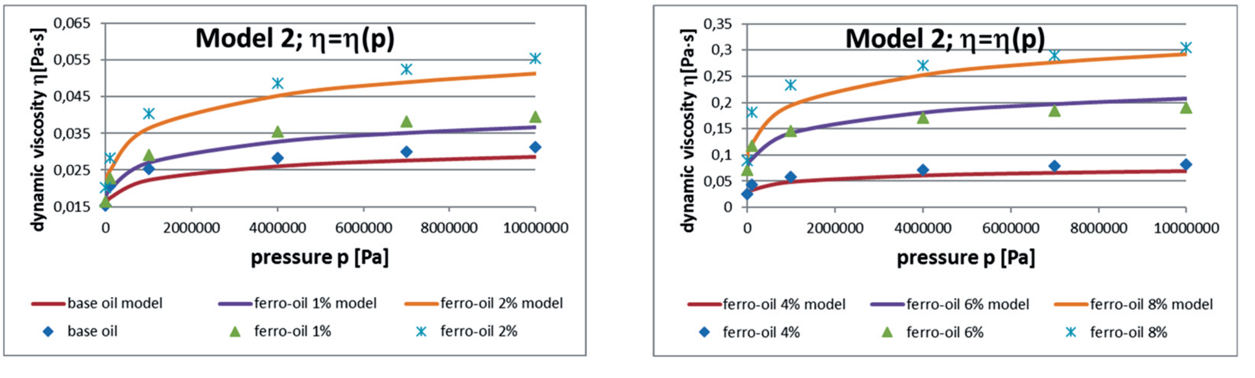

Fig. 6. Exponential model – fitted functions for the results of the viscosity changes due to the temperature for base-oil and 8% ferro-oil
Rys. 6. Model wykładniczy funkcji dopasowań – przykładowe zmiany lepkości dynamicznej z ciśnieniem dla 8% ferro-oleju oraz oleju bazowego

Table 3. The parameter values of matching function and the variance explained for Model 2
Tabela 3. Wartości parametrów funkcji dopasowania oraz wariancji wyjaśnionej dla Modelu 2

Magnetic particle concentrations	base oil	1% ferro-oil	2% ferro-oil	4% ferro-oil	6% ferro-oil	8% ferro-oil
R^2 – share of square variance	0.9718548	0.9889496	0.9936194	0.9578009	0.9577592	0.8833339
η_0 – characteristic value of dynamic viscosity	0.1547	0.01563	0.01940	0.02598	0.06551	0.09230
a_2 – proportionality ratio	0.20331	0.28924	0.36022	0.49485	0.35844	0.46821
δ_{p2} – pressure parameter	223.289	42.3649	20.1215	15.3227	29.9379	16.6094

The summary of parameter values of adjustment for the logarithmic model as well as the corresponding shares in variants are presented in **Table 3**.

Due to the complex forms of the relationships between the concentration of the magnetic particles ncs in ferro-oils and adjustment parameters of Model 2, i.e. logarithmic, no additional calculations of the form of function $\eta = \eta(n_{cs}, p)$ have been made.

Model 3: Power model has been described with the following relation (11):

$$\eta_p = \eta_0 \cdot (1 + a_3 \cdot (p)^{\delta p3}) \qquad (11)$$

where η_p – dynamic viscosity depended on pressure [Pa·s],
η_0 – initial dynamic viscosity for p_0 [Pa·s],
δ_{p3} – factor taking into account the influence of pressure p on the dynamic viscosity of the ferro-oil for the power model [-],
a_3 – proportionality factor for power model [Pa^{-1}],
p – hydrodynamic pressure [Pa].

Figure 7 represents the obtained characteristics of adjustment for the power model.

Obtained values of the adjustment parameters as well as corresponding explained variances for the power model are presented in **Table 4**.

The proposed power model in the form described with the dependency (11), almost ideally maps the

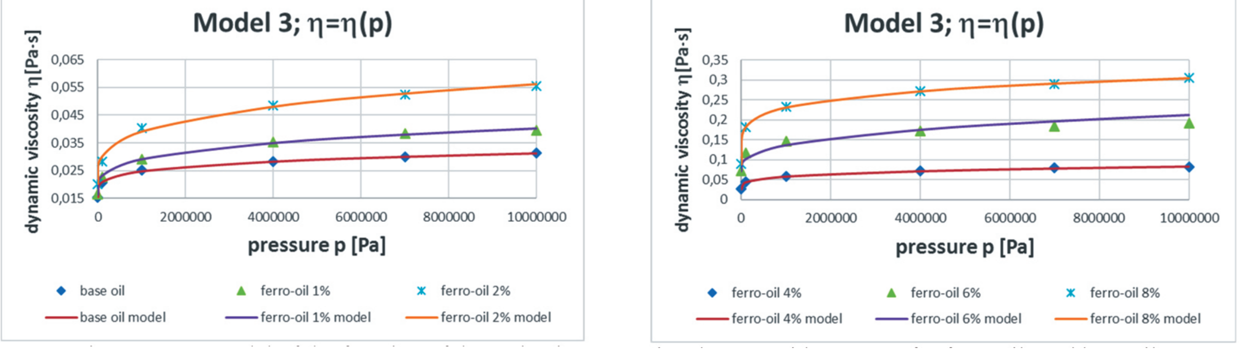

Fig. 7. Power model of the function of dynamic viscosity changes with pressure for ferro-oils and base oil
Rys. 7. Model potęgowy funkcji dopasowań zmian lepkości dynamicznej z ciśnieniem dla ferro-olejów i oleju bazowego

Table 4. The parameter values of matching function and the variance explained for Model 3
Tabela 4. Wartości parametrów funkcji dopasowania oraz wariancji wyjaśnionej dla Modelu 3

Magnetic particle concentrations	base oil	1% ferro-oil	2% ferro-oil	4% ferro-oil	6% ferro-oil	8% ferro-oil
R^2 – share of square variance	0.986273	0.990809	0.992492	0.993003	0.995301	0.994222
η_0 – characteristic value of dynamic viscosity	0.1547	0.01563	0.01940	0.02598	0.06551	0.09230
a_3 – proportionality ratio	0.9353	0.795	0.7372	0.7279	0.6887	1.6732
δ_{p3} – pressure parameter	-0.048053	-0.050638	-0.052605	-0.053772	-0.054196	-0.057493

course of the actual viscosity characteristics. The values of the explained variance R2 are very high and range between 0,986 for the base oil and up to 0,995 for the 6% ferro-oil. The precision of replication is maintained in the whole examined area, both for low and for the high values of pressure. In addition, as opposed to the logarithmic model, there are no discontinuity areas and the power relationship is characterized by the relatively high susceptibility to conducting mathematical operations in the further stages of the analytical and numerical research.

OBSERVATIONS AND CONCLUSIONS

Among the models being the subject of the analysis, the exponential model maps the actual courses of the change in the viscosity of the ferro-oils and the base oil in relation to the pressure in the weakest degree. The errors generated in this way become even more severe when a wider range of pressure is taken into account. The application of this model in the description of the process of change in viscosity with the parameter of pressure would be justifiable only by its high susceptibility to the mathematical "processing." However, it is worth noticing that, only for this particular one among the researched models of adjustment, it was relatively easy to calculate a relation (9) connecting the concentration of the magnetic particles in the ferro-oil with the change in its viscosity.

The assessment of the logarithmic model cannot be conclusive and conclusions resulting from it are of rather ambivalent nature. On the one hand, the degree of replication of the actual courses in the change of viscosity in relation to the pressure is probably, for the most reviewed cases, highly satisfying. On the other hand, the mathematical properties of the logarithmic function undermine this advantage and most probably make this logarithmic model unusable in the situation when its further mathematical transformation is required. According to the author, this excludes further use of the model in analytical and numerical research on the changes of the flow and operational parameters of the transverse slide bearing lubricated with the ferro-oils.

Therefore, the most promising of the free in the aspect of further use in research proved to be the power model, due to its relatively simple mathematical structure, strong physical references, and a very high accuracy in the replication of the actual characteristics. In addition, it is possible to calculate the dependency between the parameters of adjustment a3 and dp3 and the concentration of magnetic particles ncs in the ferro-oils with the use of the linear or exponential functions with this model, at the expense of, however, significant complexity as compared to the current, simple mathematical relation, along with all the consequences resulting from it.

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